

[illegible]

REPORT DATE

3. REPORT TYPE AND DATES COVERED

FINAL 01 April 90 to 30 Sep 90

### 3. TITLE AND SUBTITLE

# OPTIMAL MAINTENANCE STRATEGIES FOR REPAIRABLE SYSTEMS WITH GENERAL DEGREE OF REPAIR

### 5. FUNDING NUMBERS

AFOSR-90-0208  
61102F 2304/A5

6. AUTHOR(S)

DROR ZUCKERMAN

7 PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

COLLEGE OF BUSINESS AND ADMINISTRATION  
DREXEL UNIVERSITY  
32nd AND CHESTNUT STREETS  
PHILADELPHIA, PA 19104

8. PERFORMING ORGANIZATION  
REPORT NUMBER

AFOSR-TR

91

~~06/25~~  
06/29

9. SPONSORING MONITORING AGENCY NAME(S) AND ADDRESS(ES)

**APOSR/NM**

Bldg 410

Holling AFB DC 20332-6448

10. SPONSORING MONITORING  
AGENCY REPORT NUMBER

AFOSR-90-0208

## 11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT

Approved for public release  
distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Our project focuses on a single unit system which is subject to random failure. The age of the system in service is maintained as a control variable. Upon failure, an age-dependent maintenance action specifying the degree of repair is taken by a controller. By employing analytical tools and numerical procedures, we investigate and characterize the structures of the optimal repair policy under a discounted cost optimality criterion. Special analytical and numerical effort is directed throughout the study for the development of efficient computational procedures for the optimal strategy.

#### 14. SUBJECT TERMS

15 NUMBER OF PAGES

15. PRICE CODE

17 SECURITY CLASSIFICATION  
OF REPORT  
UNCLASSIFIED

18 SECURITY CLASSIFICATION  
OF THIS PAGE  
UNCLASSIFIED

19 SECURITY CLASSIFICATION  
OF ABSTRACT  
UNCLASSIFIED

20 LIMITATION OF ABSTRACT

UL

Title of Project:

**OPTIMAL MAINTENANCE STRATEGIES FOR  
REPAIRABLE SYSTEMS WITH GENERAL  
DEGREE OF REPAIR**

Principal Investigator (PI):

Dror Zuckerman, Professor

Department of Management

College of Business and Administration  
Drexel University  
32nd and Chestnut Streets  
Philadelphia, PA 19104

Electronic Mail : ZUCKERMD @ DUVN

**91-05860**



91

22 048

A-1

## TABLE of CONTENTS

		Page
1.0	SCIENTIFIC REPORT (Spring 1990)	1
1.1	Summary	1
1.2	Description of the maintenance model	1
1.3	Main findings	2
2.0	FUTURE RESEARCH CHANNELS	3
3.0	THE IMPACT OF THE PROJECT	4
4.0	DATA REGARDING THE PRINCIPAL INVESTIGATOR	6
4.1	Personal data	6
4.2	Publications in the past five years	7
4.3	Additional information	8

## 1. SCIENTIFIC REPORT

### 1.1 Summary

The study is a natural extension of the existing literature regarding maintenance policies with minimal repair.

The main objective of the research is to explore optimality issues concerning maintenance strategies with general degree of repair.

In contrast to the existing approach in the literature, we consider maintenance strategies in which the degree of repair is a decision variable determined endogenously.

Our project focuses on a single-unit system which is subject to random failure. The age of the system in service is maintained as a control variable. Upon failure, an age-dependent maintenance action specifying the degree of repair is taken by a controller.

By employing analytical tools and numerical procedures, we investigate and characterize the structures of the optimal repair policy under a discounted cost optimality criterion. Special analytical and numerical effort is directed throughout the study for the development of efficient computational procedures for the optimal strategy.

### 1.2 Description of the maintenance model

We examine a single-unit system which is subject to random failure. At any point of time the available information to a controller regarding the state of the system is composed of:

- I. The virtual age of the system.
- II. The status of the system (operating, or failed).

The system's lifetime is characterized by a known distribution function  $F(x)$ . Upon failure, an age-dependent repair action, executed in negligible time, is taking place. We consider a general repair model in which the controller may select the degree of repair in the range between minimal repair and perfect repair.

Let  $t_1, t_2, t_3, \dots$ , be the failure points of time, and let  $X(t)$  be the virtual age of the system describing its effective age. The virtual age process increases continuously at rate of 1 between failures, and it has discontinuities at repair points of time. Assuming right continuity, i.e.,  $X(t) = X(t+)$ , then

$$X(t_n) = X(t_n -) - d$$

where  $d \in [0, X(t_n -)]$  is the maintenance action employed at time  $t_n$ . In the proposed

setting, if  $d = X(t_n^-)$ , we have a perfect repair resulting in an operating system which is as good as new. If  $d = 0$ , we have minimal repair leading to a functioning system with a virtual lifetime equal to that just prior to failure.

The cost parameters associated with repair activities are defined as follows:

If  $X(t_n^-) = x$ , and a repair action  $d$  is employed, then the corresponding repair cost,  $C(x, d)$ , is composed of two factors; a repair element leading to a functioning system with the same conditions just prior to failure, and a restoration action decreasing the system's effective age to  $x - d$ .

The following reasonable and natural properties regarding the cost function are assumed:

- (a) For a given  $x$  contained in the support of  $F$ ,  $C(x, d)$  is non-negative, continuous and increasing in  $d$ , for  $d \leq x$ .
- (b) For a given repair action  $d$ ,  $C(x, d)$  is non-negative, continuous and increasing in  $x$ , for  $x \geq d$ .

The class of permissible maintenance strategies is defined as follows:

An age-dependent maintenance strategy  $\pi$  is a mapping from  $S$  to the set  $[0, \infty)$ , where  $S$  is the support of  $F$ , and  $\pi(x) \leq x$ . More explicitly, for an element  $x \in S$ , the control action  $\pi(x)$  specifies the degree of repair employed for a failed system with virtual age of  $x$ . We restrict attention to age-dependent maintenance strategies which are piecewise continuous.

The objective is to characterize and compute the class of optimal maintenance strategies which minimizes the total expected discounted cost over an infinite planning horizon. Formally, the expected discounted cost under a permissible strategy  $\pi$  can be presented as follows:

$$E \left[ \sum_{n=1}^{\infty} \exp(-\alpha t_n) C(X(t_n^-), \pi(X(t_n^-))) \right],$$

where  $\alpha > 0$ , is a continuous discount factor.

### 1.3 Main findings

By using analytical tools from the areas of Dynamic Programming, Stochastic Processes and Markov Decision Processes, we investigated key properties of:

- I The value function associated with the proposed control problem under a given permissible strategy  $\pi$ .

## II The optimal value function.

Specifically, the following results have been established:

- (a) Under any permissible maintenance strategy, the corresponding value function is continuous, and it can be presented with the aid of a differential equation.
- (b) The optimal repair action is characterized as a function of the cost parameters and the optimal value function.
- (c) Sufficient conditions under which the optimal value function is monotonically non-increasing are provided.

Using the above results and employing the Banach's fixed point theory we develop a numerical method for the computation of the value function in our control problem under a given permissible strategy  $\pi$ . The rate of convergence of the proposed numerical method is discussed.

In many practical situations, only minimal and perfect repair actions are employed. We provide a simple sufficient condition under which an optimal maintenance strategy, in the above restricted class of actions, is a control limit policy with a monotone structure.

Finally, we restrict attention to failure models with discrete lifetime distribution. By employing tools and concepts from the area of Markov Decision Processes we developed an efficient computational procedure for the optimal strategy. The proposed computational procedure has been tested for various cost functions and discrete lifetime distributions.

## 2. FUTURE RESEARCH CHANNELS

Based on the results described above regarding our basic repair model we will generalize and extend it in the following directions:

- (1) Determination of sufficient conditions on the distribution function  $F$  and the cost function  $C$  under which the optimal maintenance strategy is a bang bang strategy.
- (2) In the literature, maintenance models are divided into two distinct categories of "preventive" and "preparedness" maintenance models; preventive maintenance models assume that the state of the system is known continuously with certainty. Preparedness models on the other hand, deal with stochastically failing systems, where the state of the system is assumed to be unknown, unless either inspection or replacement is carried out. Using the above two categories we will generalize the repair model in the following directions:

(a) The basic maintenance model of our project will be explored under the assumption that failures can be discovered only by inspection. Upon detection of a failure, an age dependent repair action is employed. In this model the decision maker is interested not only in the determination of a repair policy but also in an optimal schedule of inspection activities.

(b) The action space of our maintenance model will be extended to allow a controller to institute a planned (preventive) replacement at any stopping time  $T$ .

(3) An important issue for future research is the analysis of maintenance shock models with general degree of repair. Specifically, the following general setting will be considered:

A system is subject to randomly occurring shocks. Each shock causes a random amount of damage to the system. The damage accumulates additively until failure. The time between shocks is governed by an homogeneous Poisson process. Any one of the shocks might cause the system to fail with probability which is determined by a known survivorship function. Upon failure a repair action executed in negligible time is taking place. Let  $X(t)$  be the damage level at time  $t$ , and let  $t_1, t_2, t_3, \dots$ , be the failure points of time. Then at time  $t_n$  a repair action  $d, d \in [0, X(t_n -)]$  is employed. Assuming right continuity, then the following relationship holds

$$X(t_n) = X(t_n -) - d,$$

where  $X(t_n -)$  includes the damage magnitude incurred at time  $t_n$ .

The optimal repair policy under a discounted cost criterion will be investigated analytically and numerically for a general cost function which is dependent on the state of the system and the repair action employed by a controller.

An attempt will be made to generalize the additive damage repair model to include maintenance systems with inspection and to allow a controller to institute a planned (preventive) replacement before failure.

### 3. THE IMPACT OF THE PROJECT

The continuing interest in the scientific literature in the study of maintenance models for items with stochastic failure has its roots in a wide range of applications which include maintenance of ecological balance, environmental issues, medical aspects and mainly military and industrial applications. The development of complex systems in advanced technological environment, requires improved maintenance models and control strategies.

The proposed maintenance model outlines a new research channel in the area of maintainability with interesting theoretical issues and a wide range of potential

applications in various fields such as product design, inventory systems for spare parts, and management of maintenance crews.

In order to extend the potential applicability of the model, its mathematical formulation will be presented in a sufficiently general form.

We hope that our study will stimulate additional research in the area of optimal control of repairable systems with a general degree of repair.



#### 4. DATA REGARDING THE PRINCIPAL INVESTIGATOR

##### 4.1 Dror Zuckerman - Personal data

Date of Birth: January 30, 1946  
Marital Status: Married, three children  
Telephone: (215) 895-1791

##### EDUCATION

B.Sc. - Mathematics and Statistics - The Hebrew  
University of Jerusalem, 1969  
M.Sc. - Operations Research - Technion - Haifa, 1972  
Ph.D. - Operations Research - Cornell University, 1976

##### CURRENT AND PREVIOUS POSITIONS

1988 - present	Professor, College of Business and Administration, Drexel University
1987 - 1988	Associate Professor, School of Business Administration, The Hebrew University of Jerusalem.
1985 - 1986	Associate Professor, University of British Columbia
1982 - 1984	Senior Lecturer, School of Business Administration, The Hebrew University of Jerusalem.
Summer - 1983 Summer - 1984 Summer - 1988 Summer - 1989	Research Associate, University of Toronto.
1979 - 1981	Assistant Professor, Northwestern University, Graduate School of Management.
1977 - 1978	Lecturer, Cornell University

##### ADDITIONAL ACTIVITIES

Member of the Editorial Board of Advances in management science.

## 4.2 Publications in the past five years

### 1. In the area of Reliability and Maintenance

Posner, M.J.M., Zuckerman, D., (1984) A replacement model for an additive damage model with restorstion. Operations Research Letters, Vol. 3, pp. 141-148.

Posner, M.J.M., Zuckerman, D., (1986) Semi-Markov shock model with additive damage. Advances in Applied Probability, Vol. 18, pp. 772-790.

Zuckerman, D., (1986) Optimal stopping in a continuous search model Journal of Applied Probability, Vol. 23, pp. 514-518.

Zuckerman, D., (1986) Optimal maintenance policy for stochastically failing equipment. Naval Research Logistics Quarterly, Vol. 33, pp. 469-477.

Zuckerman, D., (1989) Optimal inspection policy for a multi-unit machine. Journal of Applied Probability, Vol. 26, pp. 543-551.

Zuckerman, D., (with W. Stadje) Optimal strategies for some repair replacement models. Advances in Applied Probability. Forthcoming.

### 2. In other areas

Zuckerman, D., (1984) On preserving the reservation wage property in a continuous job search model. Journal of Economic Theory, Vol. 34, pp. 175-179.

Zuckerman, D., (1985) Optimal unemployment insurance policy. Operations Research, Vol. 33, pp. 263-276.

Ben-Horin, M., Zuckerman, D., (1987) The effect of unemployment insurance on unemployment duration. Journal of Labor Economics, Vol. 5, pp. 386-390.

Zuckerman, D., (1988) Job search with general stochastic offer arrival rate. Journal of Economics Dynamics and Control. Vol. 12, pp. 679-688.

Zuckerman, D., (1989) Capital gains taxation and the lock-in effect: A probabilistic framework. Operations Research Letters. Vol. 8, pp. 101-107.

Zuckerman, D., (with Ben-Horin) (1989) Stimulating job search through the unemployment insurance system. Operations Research. Forthcoming.

Zuckerman, D., (with M.J.M. Posner) (1990) Optimal R&D programs in a random environment. Journal of Applied Probability. Forthcoming.

#### 4.3 Additional information

In a survey published by the Institute of Management Sciences my name appears among the 50 leading productive researchers in the area of Applied Probability during the period 1980 - 1987 (see Applied Probability Newsletter (Spring 1989) Vol 13 No.1).

As suggested in the article, since it is not possible to measure objectively the quality of published research, it is reasonable instead to consider productivity in leading journals as a plausible surrogate.